Topology Optimization and Comparative Analysis of Optimal Torque and Saliency of Synchronous Machines

Alessandro Silvestrini, Maya Hage Hassan, Xavier Mininger, Guillaume Krebs, Philippe Dessante
Université Paris-Saclay, CentraleSupélec, Sorbonne Université, CNRS, Group of electrical engineering-Paris, Gif-sur-Yvette, France

Email: alessandro.silvestrini@centralesupelec.fr, maya.hage-hassan@centralesupelec.fr

***Abstract*—In this paper, we investigate two optimization problems to design the rotor of a synchronous reluctance motor (SYnRM). Both problems are based on a density-based topology optimization method. The first problem aims to maximize the average torque, while the second aims to maximize the saliency through energy formulation. The study considers different starting points that are represented through a variety of initial material distributions. The optimization algorithm is based on the gradient descent method, and the sensitivities are obtained using the adjoint variable method.**

I. INTRODUCTION

The primary objective of nearly all topology optimization (TO) focused on electric machines thus far has been to maximize the torque density or shape the torque profile. The objective function in the literature is typically formulated in terms of average torque [1]-[2], which is computed by determining its instantaneous values for different rotor positions. In principle, the average torque of a SynRM is closely linked to the saliency ratio. As proposed in [3], the saliency ratio could be expressed as the difference between the magnetic energies of the d-axis and q-axis. It is important to highlight that this method does not require the calculation of the average torque, which reduces the computation time needed to analyze different rotor positions.

This paper presents a comparative analysis of two TO problems, each formulated with distinct objective functions: one based on mean torque and the other on the saliency ratio. To address these problems, the Solid Isotropic Material with Penalization method (SIMP) is combined with the Generalized Optimality Criteria (GOC) algorithm. The nonlinear sensitivities with respect to a design variable are analytically derived using the adjoint variable method (AVM).

II. OPTIMIZATION PROBLEMS

The following methodology is applied to design the rotor of a SYnRM with a pre-designed stator. The stator consists of 36 slots, with three slots per pole and per phase and the windings are supplied with a current density J of 5 A/mm². The design domain is discretized into triangular finite elements and the total number of rotor mesh elements, N_{el}, is equal to 36033. The material property of each element e is determined by interpolating the reluctivity values of both air (ν_0) and non-linear ferromagnetic material (ν_f) with the design variable ρ of the element e such as:

$$\nu_e = \nu_0 + (\nu_f(b_e) - \nu_0)\rho_e^q \tag{1}$$

where ν_e is the element reluctivity, b_e the element magnetic flux density and ρ_e the element density. In the literature, this formulation has typically a penalty factor $q > 1$. In this case, using reluctivity, the interpolation is naturally penalized, so $q = 1$. The goal is to optimize the rotor geometry, finding the optimal material distribution. The two TO problems are formulated to maximize the average torque (2) and the saliency (3) while satisfying a total rotor maximum volume (V_{max}), fixed at 60 % to achieve a lightweight motor design. Both optimization problems are solved using the GOC algorithm [4]. Gradient-based algorithms are sensitive to the initialization points and often lead to local minima. For this reason, different initial material distributions are used to initialize the problems, as shown in Fig. 1a and Fig. 1b.

A. TORQUE AND SALIENCY MAXIMIZATION

The problem of maximizing the average torque (T_{avg}) is formulated as described in (2), while the maximization of saliency is given in (3). Both problems share the same constraints, as specified in (4).

$$\min_{\rho} \quad f_t(\rho) = -T_{avg}(\rho) = -\frac{1}{N}\sum_{k=1}^{N} T_{\theta_k}(\rho) \tag{2}$$

$$\min_{\rho} \quad f_s(\rho) = -(W_d(\rho) - W_q(\rho)) \tag{3}$$

$$\begin{aligned} \text{s.t.} \quad & \sum_{e=1}^{N_{el}} \rho_e V_e = V_{max} \\ & 0.001 \le \rho_e \le 1 \end{aligned} \tag{4}$$

The torque in (2) is calculated by evaluating N rotor positions, with 2° steps. The instantaneous values, T_θ, are computed in the airgap using Arkkio's method, with a current phase angle of 45°. In (3), to determine the two magnetic energies (W_d, W_q), windings are fed respectively by a d-axis and q-axis current with a respective current angles of 0° and 90°. Thus, the average torque is maximized through the saliency.

The sensitivities of the two objective functions with respect to the design variable are computed with the AVM and, respectively, for the torque and saliency optimization problem:

$$\frac{\partial f_t(\rho)}{\partial \rho} = \frac{1}{N}\sum_{k=1}^{N} \lambda_k^T \frac{\partial (K_m(A_{\theta_k})A_{\theta_k} - J_{\theta_k})}{\partial \rho}\bigg|_{\rho} \tag{5}$$

$$\frac{\partial f_s(\rho)}{\partial \rho} = \frac{\partial (W_d(\rho) - W_q(\rho))}{\partial \rho}\bigg|_A + \lambda_d^T \frac{\partial (K_m(A_d)A_d - J_d)}{\partial \rho}\bigg|_\rho + \lambda_q^T \frac{\partial (K_m(A_q)A_q - J_q)}{\partial \rho}\bigg|_\rho \quad (6)$$

where K_m is the magnetic stiffness matrix that contains the magnetic non-linearity of the material, A the magnetic vector potential, J is the current density vector and λ the adjoint state.

The optimization is performed with periodic filtering [5] on the density (7). This procedure avoid disturbing the optimization process at each iteration, leading to a smooth design and better performance.

$$\tilde{\rho}_e = \frac{\sum_{i \in N_e} w(x_i)\rho_i v_i}{w(x_i)v_i} \quad (7)$$

Where N_e represents the neighborhood of the element e, ρ_i the element density of the neighbor i, v_i the volume of the element i, and $w(x_i)$ a weighting function that depends on the distance between the elements i and e.

III. RESULTS

The results shown in Fig. 1 summarize the optimal geometries obtained through the two problems. The torque maximization problem involves 45 Finite Element Analyses (FEA), while saliency maximization requires only 2 FEAs.
Those from torque formulation present intermediate materials, and density projection is a simple solution to remove them. Applying the projection to geometries of Fig. 1e and Fig. 1f, the mean torque is kept unchanged but the saliency values are reduced (Table I).
To compare the two formulations, the mean torque for the saliency maximization problems is computed at various stages of the algorithm. This can be observed in Fig. 2, the average torque is also optimized by maximizing the saliency. The final maximum mean torque values were obtained for torque and saliency formulation, respectively: 7.07 N.m and 6.87 N.m when initializing with holes (Fig. 2a), then 6.63 N.m and 6.83 N.m when initializing with homogeneous density material (Fig. 2b). All the results are summarized in Table I.

Table I
SALIENCY (W_d/W_q) AND TORQUE COMPARISON

Initialization	Optimization problem	Saliency Value	Torque [N.m]
Holes	Saliency	3.26	6.87
Homogeneous	Saliency	3.20	6.83
Holes	Torque	3.94	7.07
Homogeneous	Torque	3.22	6.63
Holes	Torque and projection	2.95	7.08
Homogeneous	Torque and projection	2.52	6.62

In conclusion, comparing the two formulations, the optimization considering the saliency maximization decreases the calculation time by only needing two finite element analyses per iteration. This formulation allows smooth-shaped barriers without restricting the mesh design to a single pole, unlike literature methods that impose symmetries, providing greater design flexibility.

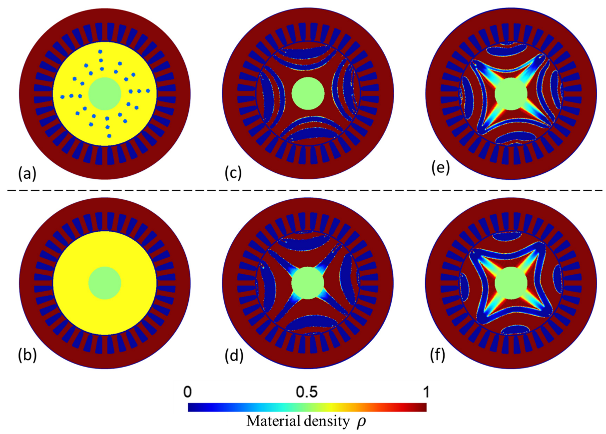
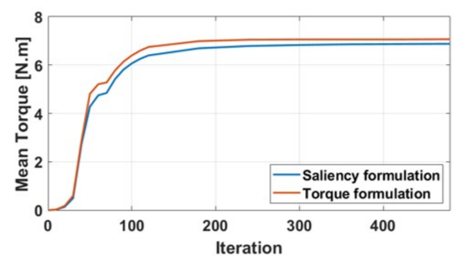
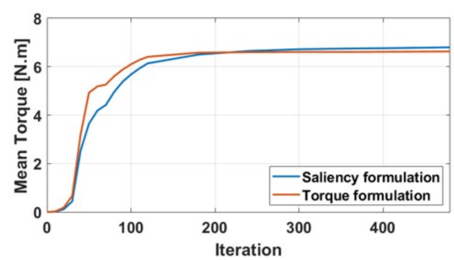


Figure 1. Initialization (a, b) and corresponding optimization results from saliency formulation (c, d) and torque formulation (e, f).



(a) Hole initialization



(b) Homogeneous initialization

Figure 2. Mean torque comparison between torque and saliency maximization problems.

REFERENCES

[1] C. Lee, and I.G. Jang, "Topology optimization of multiple-barrier synchronous reluctance motors with initial random hollow circles," Struct Multidisc Optim 64, pp. 2213–2224, 2021.
[2] O. Korman, M. Di Nardo, M. Degano, and C. Gerada, "On the Use of Topology Optimization for Synchronous Reluctance Machines Design," Energies, 15, no. 10: 3719, 2022.
[3] A. Silvestrini, M. H. Hassan, X. Mininger, G. Krebs, and P. Dessante, "Magneto-Mechanical Topology Optimization with Generalized Optimality Criteria," IEEE Transactions on Magnetics, pp.1-4, 2023.
[4] N. H. Kim, T. Dong, D. Weinberg, and J. Dalidd, “Generalized optimality criteria method for topology optimization,” Applied Sciences, vol. 11, no. 7, 2021.
[5] T. Cherrière, S. Hlioui, L. Laurent, F. Louf, H. Ben Ahmed, and M. Gabsi, "Effects of Filtering and Current-angle Adjustment on the Multi-Material Topology Optimization of a 3-phase Stator," IEEE Transactions on Magnetics, pp. 1-1, 2023.