Complex-frequency approaches for parallel computations of the eddy-current transient response

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***Abstract*—Two different approaches based on numerical inversion of the Laplace domain response are compared for the calculation of the eddy-current transient response to a coil excitation. The complex-frequency spectrum approach, fully parallelisable, proves to be better suited for the solution of the problems with high dissipation than the well-known fast Fourier transform method. The two approaches demonstrate different performances in short and long time regimes in terms of accuracy.**

I. Introduction

The calculation of transient field response in eddy-current problems involving arbitrary excitation is conventionally carried out by direct integration of the governing differential equation of diffusion via a time-stepping scheme. Such approaches comprise single or multistep background differentiation (BDF) schemes or the Runge-Kutta method. This approach is well-established and has been successfully applied in combination with discretisation methods based on volume meshes such as the finite elements method (FEM). Nevertheless, their application is not straightforward when combined with other type of methods, including integral equation methods like the boundary element method (BEM) [1] or modal approaches [2]. As far as the latter is concerned, the conventional surface development bases must be complemented by an appropriate volumetric basis in order to account for the solution of the previous time-steps, a generalisation which complicates the solution, in particular when the geometry departs from the simple multilayer pieces.

A second drawback of the time-stepping approaches is that they are in principle sequential schemes[1]. An alternative to direct time-integration are frequency domain (FD) methods, which are based on a spectrum discretisation. Since the unit calculation per frequency is described by the Helmholtz equation, these approaches are directly applicable to methods like the BEM and the mode-matching, originally developed for use in the frequency domain. Moreover, their parallelisation is straight-forward since the unit single-frequency solutions are independent to each other. The most well known spectral approach is the fast Fourier Transform (FFT), which involves the problem solution for a number of real frequencies. Nonetheless, the determination of the optimal spectrum discretisation for the diffusion problem turns to be very challenging [3]. In contrary, approaches based on a sampling in the Laplace or Z plane prove to be more robust for problems with high dissipation as the eddy-current problem. Common point of these approaches is that the sample frequencies are in general located in the complex plane . Yet these methods also have their limitations since their robustness depend on the type of excitation and they are in general less accurate for long-time computations.

In this contribution, two alternative approaches based on the inversion of the Laplace and Z transform will be compared for the solution of the eddy-current problem under step excitation. This type of excitation is important since it corresponds to the majority of excitation signals used in pulsed eddy-current applications, and because, once the response under step excitation is known, one can easily compute the corresponding signal under arbitrary excitation using Duhamel's integral [2]. The two approaches will be compared against the results of first order BDF schemes (BDF1), our reference. Long-time behaviour will be studied and accuracy improvement recipes will be tested.

II. Time-domain formulation

The underlying physical problem is described by the diffusion equation for the magnetic vector potential $\mathbf{A}$:

$$\nabla \times \nu \nabla \times \mathbf{A} + \sigma \frac{d\mathbf{A}}{dt} = \mathbf{J} \tag{1}$$

where ν, σ is the magnetic reluctivity and the electrical conductivity of the medium, respectively, and $\mathbf{J}$ the excitation current. The magnetic vector potential is defined in terms of the magnetic flux density via the relation

$$\mathbf{B} = \nabla \times \mathbf{A}. \tag{2}$$

Since $\mathbf{A}$ is defined up to a gradient, it must be gauged. The usual choice is to impose a zero gradient condition, which is known as the Coulomb gauge.

For the integration of the diffusion equation we introduce a homogeneous time discretisation $t_i = i\Delta t$, $i = 0, 1, \ldots, N_t$, and we apply a BDF1 scheme, which yields the following recursive formula

$$\left(\nabla \times \nu \nabla \times + \frac{\sigma}{\Delta t}\right) \mathbf{A}_i(\mathbf{x}) = \frac{\sigma}{\Delta t} \mathbf{A}_{i-1}(\mathbf{x}) + \mathbf{J}_i(\mathbf{x})\,. \tag{3}$$

Alternatively the transient solution can be retrieved by inversion of N_f independent monochromatic problems with complex frequencies in the complex plane.

[1]Parallel-in-time time-stepping schemes are a subject of current research, yet the proposed approaches are more complicated than their classical sequential counterparts both in terms of implementation and stability considerations.

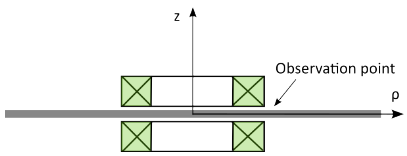

Fig. 1. Problem configuration. Infinite conducting plate excited by a cylindrical coil with normal to the interface axis. The field is calculated at a point at the opposite side of the plate.

III. Solution in the complex frequency plane

A. Evaluation via numerical inversion of the Z transform

Applying the Z-transform to (3), we yield

$$\left(\nabla \times \nu \nabla \times + \sigma \frac{1 - z^{-1}}{\Delta t}\right) \mathbf{A}(\mathbf{x}, z) = \mathbf{J}(\mathbf{x}, z)\,. \qquad (4)$$

A straight forward method for inverting (4) back into the time domain is by numerical contour integration along the unit circle.

Choosing a discretisation of the integration contour $z_k = e^{2\pi i \frac{k}{N_f}}$ with $k_f = 1, \ldots, N_f$ and applying the trapezoidal rule for the numerical computation of the integral, we obtain [4]

$$\mathbf{A}_n(\mathbf{x}) = \frac{1}{N_f} \sum_{k=1}^{N_f} \tilde{\mathbf{A}}(\mathbf{x}, z_f)\, z_f^n \qquad (5)$$

where $\tilde{\mathbf{A}}(\mathbf{x}, z_k)$ stands for the solution of (4) at the sampling points z_k.

B. Numerical computation of solution samples using the Zakian-type methods

Another family of methods for the computation of transient signals from Laplace transformed solutions, are the Zakian-type methods. The main idea here consists in computing approximations of the delta function via exponentials and calculating the convolution with the sought time function, which reduces to the computation of a weighted sum of function samples in the Laplace plane. One of the most efficient of these formulas is the so-called Gaver-Stehfest formula [5]

$$\mathbf{A}_n(\mathbf{x}, t) \approx \ln \frac{2}{t} \sum_{n=1}^{N} K_n \tilde{\mathbf{A}}\left(\mathbf{x}, n \ln \frac{2}{t}\right) \qquad (6)$$

where $\tilde{\mathbf{A}}$ stands for the Laplace transform of the potential solution calculated at the imaginary frequencies $-i \ln 2/f$ $\ln 2/f$. The weighting coefficients K_n are tabulated numbers which depend on the order of the scheme we wish to apply.

IV. Results

As an example we consider the calculation of the transient eddy-current field in an infinite plate induced by a cylindrical coil positioned above the plate with its axis normal to the plate interface as shown in Fig. 1.

The magnetic field is calculated at a sample point underneath the plate using three approaches: direct integration with BDF1, calculation in the Z plane and inversion in the Laplace plane using the Stehfest method. The three solutions are shown as function of time in Fig. 2. The BDF1 solution used as reference has been calculated using the approach described in [2]. Both spectral solutions have been obtained by applying the same modal solver of [2] but this time solving the Helmholtz equation at the sample complex frequencies. The number of frequencies used for both Stehefest and inverse Z-transform (IZT), approaches are $10N_t$, N_t being the number of timesteps where the transient response is evaluated[2]. We observe that the two spectral methods perform differently for short and long times with the Stehfest method being more precise than CQ at later times and vice versa for the short time response.

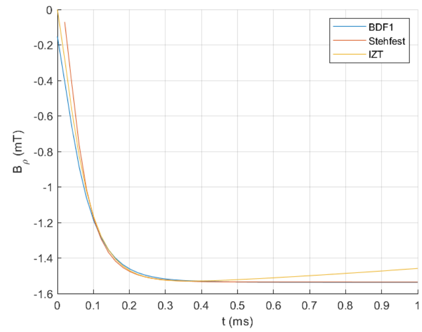

Fig. 2. Radial (B_ρ) and axial (B_z) magnetic flux density at the observation point as function of time. The transient response calculated with the Stehfest and the IZT approaches are compared against the BDF1 solution.

V. Conclusions

Two different methods based on the solution of the Helmholtz problem with complex frequencies are compared. The approach is applicable to FD solvers without modifications and allow a seamless parallelisation.The different short and long time performances indicate the need of adapted schemes for improving accuracy at the two asymptotic parts of the solution, which will be a subject of more detailed study in the main article.

References

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[2] These numbers are indicative only since the time samples used with the Stehfest method can be inhomogeneously distributed along the computation interval.