Falling magnetizable bead in a Newtonian fluid

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***Abstract*—The trajectories of a magnetic bead in a fluid subjected to the magnetic field of a ring magnet are studied. The drag and magnetic forces are computed, and the positions versus time are found, and compared with experimental ones. Due to the magnetic force, a vertical motion followed by a lateral motion, makes possible the analysis of the vertical drag force, and the lateral one.**

I. Introduction

The classic falling-ball viscometer is used in the case of Newtonian fluid and the bead has vertical and creeping motion. If a ring magnet is added (for ferromagnetic beads), an equilibrium position is found : it allows more repeatable experiments. A lateral motion also happens : the wall-effects of the cylinder may be considered separately.

The aim of the paper is to consider the trajectories of a falling bead in a Newtonian fluid when subjected to a magnetic field of a ring magnet, from a computational and experimental point of view.

II. Trajectories computation

Magnetic fields are commonly used in fluid problems to modify the motion of a ferromagnetic bead [1]. The magnetic field is created here by a ring magnet (Fig. 1), and the motion of the ferromagnetic bead (of position $\vec{x}$) in a cylinder filled with fluid is considered :

$$m\frac{d^2\vec{x}}{dt^2} = \vec{f}_d + \tilde{m}\vec{g} + \vec{f} \tag{1}$$

where m is the mass, $\vec{f}_d$ the drag force, $\tilde{m}$ the modified mass due to buoyancy, $\vec{g}$ the gravity and $\vec{f}$ the magnetic force. The motion is both vertical (along z) and lateral (along $r = \sqrt{x^2+y^2}$).

For Newtonian fluids, when the radius of the cylinder containing the fluid is infinite, the Stokes formula gives the drag force as :

$$\vec{f}_d = -6\pi\mu_d R_b \frac{d\vec{x}}{dt} \tag{2}$$

where μ_d is the dynamic viscosity, R_b the bead radius. If the cylinder has a finite radius R_t, and if the bead stays on the axis of the cylinder ($r = 0$) the Francis correcting factor is added to (2):

$$k_F = \left(\frac{1 - 0.475 R_b/R_t}{1 - R_b/R_t}\right)^4 \tag{3}$$

When the initial position is not $r = 0$, wall effects have to be included. The 3D Stokes problem has to be computed to find the real drag force of the bead in the fluid cylinder. Then additional correcting factors α_r and α_z, depending on

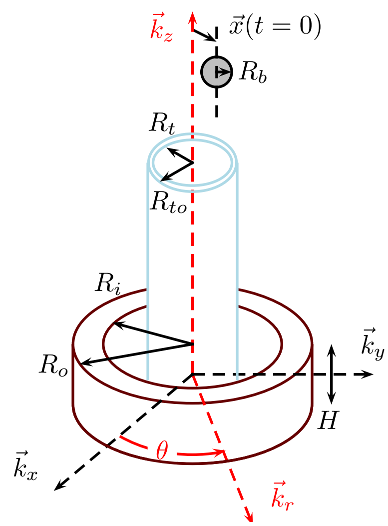

Figure 1. Sketch of falling bead

(r, R_b, R_t) are added to the Stokes formula (2) respectively for lateral and vertical motions. The increase of the coefficients when the bead is close to the wall has to be taken into account to get the real trajectory. The case of non-Newtonian fluids may also be considered, but is beyond the scope of the short paper.

The system of ordinary differential equations which describes the trajectory is :

$$\frac{d}{dt}\begin{bmatrix} r \\ m\dot{r} \\ z \\ m\dot{z} \end{bmatrix} = \begin{bmatrix} \dot{r} \\ f_r(r,z) - \alpha_r\, 6\,\pi\, R_b\, \mu_d\, \dot{r} \\ \dot{z} \\ f_z(r,z) - \alpha_z\, 6\,\pi\, R_b\, \mu_d\, \dot{z} - \tilde{m}\, g \end{bmatrix} \tag{4}$$

The aim of the paper is to find the forces as functions of the position, and then to compute the trajectories, and compare them to experimental ones, to find the fluid properties and to study the wall-effects.

III. Magnetic force computation

When ferromagnetic objects are introduced, in the neighbourhood of a magnet, of source field $\vec{h}_s$, an induced field is created, and the total field is the sum of the source field and the induced field. Then total induction verifies :

$$\vec{b}(\vec{x}) + \vec{b}_s(\vec{x}) = \mu(\vec{x})\ (\vec{h}(\vec{x}) + \vec{h}_s(\vec{x})) \tag{5}$$

where $\vec{b}$, $\vec{h}$, μ are respectively the induced induction, the induced magnetic field, and the permeability. The force which acts on the bead, can be obtained by virtual work : the virtual displacement of the ferromagnetic domain is considered to find the force due to the magnet. However, if the ferromagnetic domain moves, then the permeability function changes, which is not the case if the magnet is moved instead. Then, for reasons of simplicity, the choice is made to move the magnet from its initial position labelled 1 with a translation $-\delta\vec{X} = -\delta z\,\vec{k}_z$ to a new position labelled 2. The induced magnetic fields corresponding to both positions 1 and 2 are labelled $\vec{h}_1$ and $\vec{h}_2$. The field difference between $\vec{h}_1$ and $\vec{h}_2$ verifies :

$$\nabla\cdot\Big[\mu(\vec{x})\,(\vec{h}_2(\vec{x})-\vec{h}_1(\vec{x})) + (\mu(\vec{x})-\mu_0)\,(\vec{h}_s(\vec{x}+\partial\vec{X})-\vec{h}_s(\vec{x}))\Big] = 0 \quad (6)$$

The difference of source magnetic fields can be expressed as a Taylor series expansion (up to the second order) as :

$$\vec{h}_s(\vec{x}+\partial\vec{X})-\vec{h}_s(\vec{x}) \simeq \vec{\vec{\nabla}}\vec{h}_s(\vec{x})\,\delta\vec{X} + \frac{1}{2}\delta\vec{X}\,\vec{\vec{\vec{\nabla}}}\vec{h}_s(\vec{x})\,\delta\vec{X} \quad (7)$$

The field difference between $\vec{h}_1$ and $\vec{h}_2$ is :

$$\vec{h}_2-\vec{h}_1 \simeq \vec{\vec{\nabla}}\vec{h}_1(\vec{x})\,\delta\vec{X} + \frac{1}{2}\delta\vec{X}\,\vec{\vec{\vec{\nabla}}}\vec{h}_1(\vec{x})\,\delta\vec{X} \quad (8)$$

Since the following relationship is verified (D is the domain of the bead) :

$$\int_D (\mu(\vec{x})-\mu_0)\;(\vec{h}_s(\vec{x}+\delta\vec{X})-\vec{h}_s(\vec{x}))\cdot\vec{h}_1(\vec{x})\,d\vec{x} = \int_D (\mu(\vec{x})-\mu_0)\;\vec{h}_s(\vec{x})\cdot(\vec{h}_2(\vec{x})-\vec{h}_1(\vec{x}))\,d\vec{x} \quad (9)$$

The force components can then be computed as :

$$f_i = \int_D (\mu(\vec{x})-\mu_0)\,\vec{h}_s(\vec{x})\cdot\partial_i\left(\vec{h}(\vec{x})+\vec{h}_s(\vec{x})\right)\,d\vec{x} = \int_D (\mu(\vec{x})-\mu_0)\,\left(\vec{h}(\vec{x})+\vec{h}_s(\vec{x})\right)\cdot\partial_i\vec{h}_s(\vec{x})\,d\vec{x} \quad (10)$$

where i stands for (x,y,z), and $\vec{h}=\vec{h}_1$ is the field of the initial position, and the fields $\partial_i\vec{h}$ are computed by :

$$\nabla\cdot\Big[\mu(\vec{x})\,\partial_i\vec{h}(\vec{x}) + (\mu(\vec{x})-\mu_0)\,\partial_i\vec{h}_s(\vec{x})\Big] = 0 \quad (11)$$

The components of the stiffness matrix are :

$$\frac{\partial f_i}{\partial j} = \int_D (\mu-\mu_0)\left[\begin{array}{c}\left(\vec{h}+\vec{h}_s\right)\cdot\partial_{ij}\vec{h}_s+\partial_i\vec{h}_s\cdot\partial_j\vec{h}_s\\ +\frac{1}{2}\left(\partial_i\vec{h}_s\cdot\partial_j\vec{h}+\partial_j\vec{h}_s\cdot\partial_i\vec{h}\right)\end{array}\right]d\vec{x} \quad (12)$$

where i and j stand for (x,y,z). The source field $\vec{h}_s(\vec{x})$ of the ring magnet has a closed-form expression for a constant magnetization, so its derivatives cans be easily computed. If the magnetization is not in the axis of the ring, then more elaborate formulas may be found [2]. The magnetic force has to be pre-computed as a function of (r,z) to use it afterwards for the ordinary differential set of equations (4). For a given mesh of (r,z) positions, the use of splines with force derivatives allow a better accuracy.

IV. Results

A ferromagnetic bead (here of radius $R_b = 1.45mm$) is dropped in the tube filled with glycerol ($\mu_d = 1$ Pa.s). The magnetization of the ring magnet is $M = 0.88\,10^6 A/m$. The ball falls almost at constant r at the beginning and then at almost constant z dividing the trajectory into two main motions (Fig. 2). First, the magnet attracts the bead for a height being approximately the equilibrium position. Then, the radial motion occurs until the bead stops as it reaches the tube wall. Up to 0.5s the position is close to the free fall one. Two cameras get experimental (x,z) and (y,z) positions (crosses on Fig. 2) as a function of time. The position $r=z=0$ is the center of the magnet. The accuracy of the computed trajectory is quite good, and validates the model.

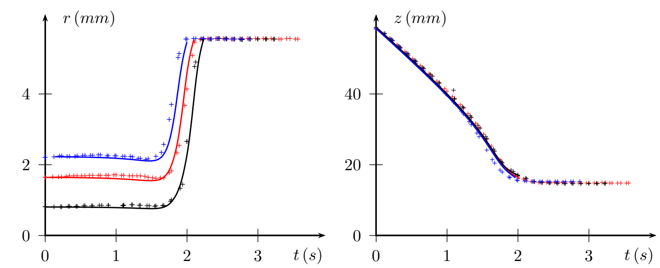

Figure 2. Position of a bead as a function of time (crosses : experimental, plain lines : numerical) for different starting abscissa positions.

The computed velocities are also close to the experimental ones. The free-fall speed (about 18mm/s) is reached in 0.1s, and then the bead accelerates due to the magnetic force, and decelerates to reach the equilibrium position.

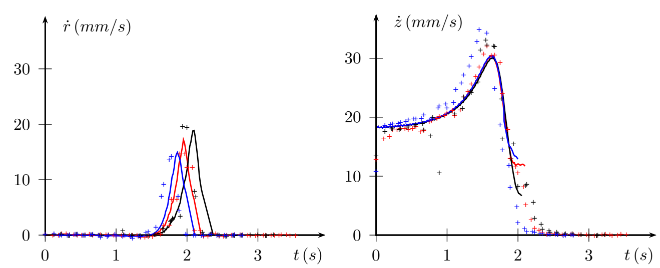

Figure 3. Bead velocities as a function of time (crosses : experimental, plain lines : numerical) for different starting abscissa positions.

The experimental device allows to find the fluid viscosity by reproducing numerically the experimental trajectories. The lateral motion also gives the order of magnitude of the drag force with wall effects.

References

[1] O.A. Sharova, D.I. Merkulov et al, "Motion of a spherical magnetizable body along a layer of magnetic fluid in a uniform magnetic field"*Physics of Fluids* 33, 087107, 2021.

[2] A. Caciagli, R. Baars, A.P. Philipse, B.W. Kuipers "Exact expression for the magnetic field of a finite cylinder with arbitrary uniform magnetization", *Journal of Magnetism and Magnetic Materials*, 456. pp. 423–432, 2018.