# Advanced optimization strategy to design efficient sub-wavelength deflectors using statistical learning

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***Résumé*—A new approach for inverse design is proposed to optimize a deflector structure, composed of sub-wavelength dielectric pillars, for Ka-band antenna applications. This numerical methodology involves two essential elements : on the one hand, a high order hybridized Discontinuous Galerkin Frequency-Domain solver of Maxwell equations combined with a Floquet modes decomposition to numerically analyze the periodic macrocell configuration of the deflector, on the other hand, a statistical learning-based global optimization algorithm to uncover the geometric properties of the dielecteric pillars, guided by a specific design objective. In this scenario, the deflector is tailored for optimal performance in Ka-band, achieving 31° deflection angle at 29 GHz. The optimization results reveal significant improvements, notably in boosting the diffraction efficiency and reducing the side lobe levels.**

## I. DESIGN METHODOLOGY

The objective of this work is to develop an advanced numerical methodology for the design of periodic sub-wavelength deflectors composed of macrocells repeated in two dimensions along x- and y- axis. Our recent efforts are focused on achieving this objective through the utilization of two leading numerical methods : a high-order discontinuous finite element method for simulating the electromagnetic problem, and a global optimization technique based on statistical learning.

### A. Numerical method

A full-wave approach is used to characterize the deflector by solving the complete system of 3D frequency-domain Maxwell equations. For this purppose, we exploit an in-house Hybridized Discontinuous Galerkin Frequency-Domain (HDGFD) solver that we have recently developed [1], [2] and which is implemented in the DIOGENeS software suite [3]. The solver is programmed in Fortran 2008 and is tailored for high-performance computing systems.

The grating structure, with cylindrical sub-wavelength pillars, is then modeled as an infinite periodic structure with periodic boundary conditions (PBC). Therefore, we can describe how the transmitted electromagnetic field is distributed into diffraction orders by using Floquet modal decomposition theory, which in the present case amounts to define and compute the mode diffraction efficiencies $\eta_{m,n}$ in Ka-band frequency range. An illustration of the periodic macrocell of the deflector simulated by the HDGFD method is shown in Figure 1. A linearly polarized plane wave is used as incident wave, excited at the lower Floquet port. Hence, the diffraction efficiencies are computed at the upper Floquet port. The objective is to enhance the diffraction efficiency of the (0,-1) mode order that corresponds to the desired deflection angle, while concurrently minimizing the efficiency level for other diffraction modes.

### B. Optimization method

An inverse design problem is formulated to optimize the deflector's macrocell structure. In the present case, the diffraction efficiency $\eta_{0,-1}$ in the desired diffraction order (0,-1) and for the intended linear polarizations (TE/TM) is used to define our objective function. This inverse problem aims to determine the positions and dimensions of cylindrical pillars to minimize the cost function. Solving this inverse problem requires a rigorous and computationally efficient optimization strategy, which is capable of handling several design parameters while minimizing the number of full-wave solver calls.

We consider here one of the most advanced optimization techniques based on a statistical learning approach, known as the Efficient Global Optimization (EGO) method [4], [5], which belongs to the class of Bayesian optimization methods. The EGO method is a global optimization algorithm that substitutes the complex and costly iterative electromagnetic evaluation process with a simpler and cheaper model. Contrary to the traditional global optimization strategies like Genetic Algorithms, the EGO method is not based on adaptive sampling, but on a surrogate model to built on a set of available fitness observations. In our case, we use Gaussian Process (GP) which, due to its statistical nature, provide both a prediction of the cost function and a measure of the uncertainty of the prediction. This surrogate model utilizes a statistical learning criterion related to the optimization target (usually called merit function) in order to identify which design should be tested in the next iteration that would provide better results close to the predefined goal.

In practice, the EGO is based on two phases. The first one is the Design Of Experiment (DOE), in which an initial database is generated. In essence, a Latin Hypercube Sampling (LHS) strategy is deployed in order to generate different designs in which the cost function is evaluated using our electromagnetic HDGFD solver. In the second phase, a GP model is constructed to fit the data obtained from DOE. The GP mean provides the surrogate model that we use to approximate the objective function and the GP variance represents the uncertainty of the model. Next, a statistical merit function is utilized to identify what areas in the domain should be targeted for

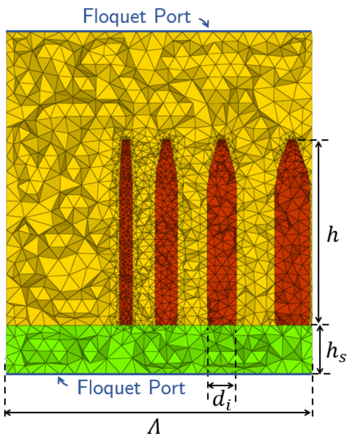

FIGURE 1. Design configuration of the HDGFD full-wave simulation.

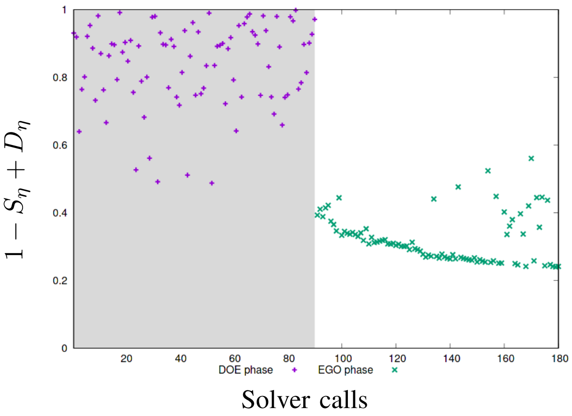

FIGURE 2. Optimization using the EGO method. Evolution of the objective function with the number of full-wave solver calls.

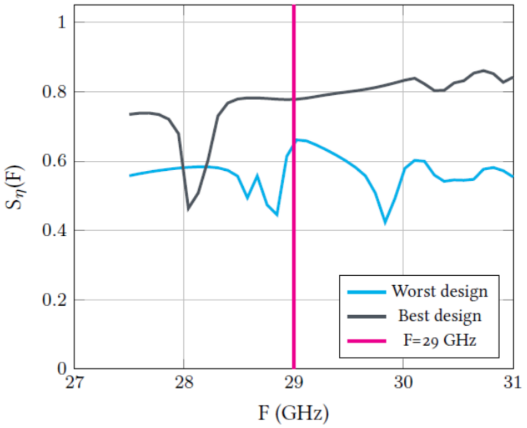

FIGURE 3. Spectra $S_\eta$ of the worst and best designs of the EGO phase.

exploitation and which should be explorated. In our case, we rely on the expected improvement, which estimates the expected progress in the cost and whose maximum defines the next design parameters set to be evaluated using the HDGFD solver. Then the database is updated accounting for this new observation (construction of a new GP model based on the updated database). We repeat this process until a predefined convergence criterion is reached, or when the expected improvement is sufficiently small.

## II. NUMERICAL RESULTS

### A. *Simulation configuration*

The objective is to optimize the design of a sub-wavelength deflector within the frequency range [27.5 GHz, 31 GHz]. The period of the macrocell along y-axis is given by $P_y = \Lambda = 20\,\text{mm}$ and the period along x-axis is chosen $P_x = 2.6\,\text{mm}$. The substrate thickness is 3 mm. The excitation used is a plane wave with linear TE/TM polarization. The design parameters to be optimized include the diameters $d_i$ and height $h$ of the pillars : $d_i \in [0.8\,\text{mm},\ 2.4\,\text{mm}]$ and $h \in [6\,\text{mm},\ 15\,\text{mm}]$ with $i \in \{1, .., 4\}$, leading to 5 optimized parameters. For a linear polarization, that combines both TE and TM polarizations, the objective is to maximize the sum $S_\eta = \left(\eta(0,-1)_{TE} + \eta(0,-1)_{TM}\right)/2$ and minimize the difference $D_\eta = |\eta(0,-1)_{TE} - \eta(0,-1)_{TM}|/2$, where the $\eta$'s are averaged in the target frequency band. Thus, the optimization problem can be written as :

$$\text{Minimize} \quad f_\eta(\mathbf{d}, h) = 1 - S_\eta + D_\eta. \tag{1}$$

### B. *Macrocell optimization results*

As explained previously, the optimization has been realized using the EGO method. During the first phase, we build a DOE database with 90 points. In the second phase, i.e., the EGO phase, a surrogate model is constructed based on these DOE designs and is refined during the optimization process to find a global minimum. Figure 2 summarizes the optimization process, in which the objective function evaluations of the DOE are displayed with purple points and the evaluations obtained during the EGO phase are displayed with green points. The deflector designs have been evaluated using the HDGFD-$\mathbb{P}_2$ method where $\mathbb{P}_2$ refers to the second-order polynomial interpolation used for the DG discretization.

The spectra of the quantity $S_\eta$ are represented in Figure 3. A comparison is depicted between the worst and the best designs of the EGO phase (white part of Figure 2). The best design achieves a notably high diffraction efficiency, reaching approximately 80% from 28.5 GHz to 31 GHz compared to a value lower than 60% for the worst design of the EGO phase, using only 180 fullwave simulations.

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