Efficient modeling of discontinuities in waveguide structures with metamaterial walls using mode-matching technique

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***Abstract* — A new methodology is proposed to model a double transition between a circular waveguide with perfect electric conducting (PEC) walls and a circular waveguide with metamaterial walls. This numerical methodology based on the Mode-Matching (MM) technique has been developed to characterize this junction and to determine the S-parameters at the level of the discontinuities by replacing the structure with an equivalent circuit. The model has been validated for a case of waveguide with metamaterial walls type 'PIC' in comparison with a full-wave FEM approach via the commercial software Ansys-HFSS™ .**

I. METHODOLOGY

A. Introduction

The use of metamaterial walls [1] offers more degrees of freedom in controlling electromagnetic field's properties within waveguides. Regarded as inhomogeneous sub-wavelength elements and arranged in repeating patterns, metamaterial introduce special effects on waveguide walls, owing to their non-linear dispersive behavior. In a spatial context, these distinctive properties of metamaterials can be used to enhance the performance of horn antennas embedded in satellites [2], which can be modeled as an alignment of waveguides with a progressively increasing cross-section. However, there are still aspects of design that require further refinement, specifically the need for rigorous analysis to master electromagnetic (EM) field distribution, within metamaterial waveguide structures with discontinuities.

For this purpose, a numerical strategy based on the Mode-Matching (MM) method [3] is used to solve this type of discontinuity problems. The main idea of this technique is to express the EM field in terms of the existing modes in the regions on both sides of the junction. The continuity condition of the transverse components of the EM field, at the discontinuity opening, leads to a system of equations that allows the determination of the reflection and transmission coefficients for hybrid modes into the structure. Nevertheless, in waveguide of complex nature, the identification of modes is difficult and the exploitation of numerical methods becomes necessary [4]. In this context, a hybrid Mode-Matching/Finite Elements (MM/FEM) approach is used to design efficient microwave structures composed of a succession of different waveguide with complex walls. Thus, in our case, we present a new methodology based on MM/FEM to analyze and characterize the propagation of EM waves in the waveguides with metamaterial walls presenting discontinuities.

B. Structure modeling

The transition between waveguides of different nature can be modeled by an equivalent circuit. The structure design studied here is a circular double transition: from a metallic waveguide to a waveguide with metamaterial walls of length L_M, then a transition to a metallic waveguide, as shown in Figure 1.

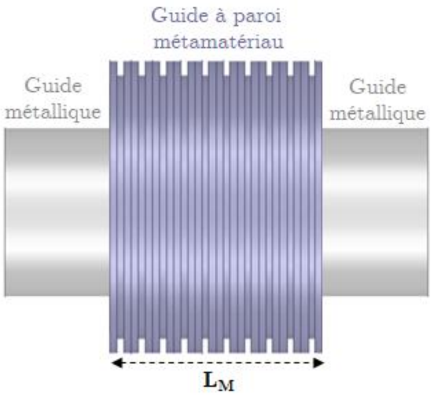

Fig. 1. Structure of the studied double transition.

Thus, based on the nature of the structure, the boundary conditions and the symmetries of geometry, the equivalent circuit can be determined [5]. In this case, the structure has a symmetry plane in the middle of the metamaterial waveguide. Therefore, it can be analyzed as two sub-problems by replacing the symmetry plane by an electric wall (odd case), and then by a magnetic wall (even case). The double transition is represented by the following equivalent circuit in figure 2.

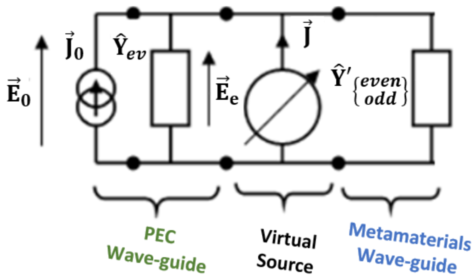

Fig. 1. Equivalent electrical circuit

The PEC waveguide is excited by the fundamental mode TE_{11} into the single-mode frequency band. It is represented by a Norton source $\vec{J}_0$, which is written in terms of the modal function $\overrightarrow{f_{11}^{TE}}$. Then, the other higher modes are all evanescent and they are characterized by the admittance operator $\hat{Y}_{ev}$. At the discontinuity, the virtual source $\vec{E}_e$ represents the passage conditions at the junction between the two waveguides. On the other hand, the admittance operator $\hat{Y}'_{\begin{Bmatrix} even \\ odd \end{Bmatrix}}$ represent the behavior of the hybrid modes in the waveguide with metamaterial walls for the two symmetry cases.

Kirchhoff's circuit laws are used to solve this equivalent circuit. Thus, the sources are related to their dual sources according to a linear system. Using the Galerkin method, the system is projected onto the excitation mode TE_{11} of the PEC waveguide. This system allows to find the admittance at the discontinuity interface by the following equation:

$$Y_{\substack{even\\odd}} = \sum_{\substack{m,n=1\\ \alpha=+,-}}^{\infty} \left\langle \overrightarrow{f_{11}^{TE}} \middle| [Y'^{\alpha}_{mn}] \overrightarrow{f'^{\alpha}_{mn}} \right\rangle \left\langle \overrightarrow{f'^{\alpha}_{mn}} \middle| \overrightarrow{f_{11}^{TE}} \right\rangle \begin{cases} th\left(\dfrac{\gamma'^{\alpha}_{mn} L_M}{2}\right) \\ coth\left(\dfrac{\gamma'^{\alpha}_{mn} L_M}{2}\right) \end{cases} \quad (1)$$

where $\overrightarrow{f'^{\alpha}_{mn}}$ are the hybrid modes of the waveguide with metamaterial walls and $[Y'^{\alpha}_{mn}]$ are their characteristic impedances. The modal proprieties are determined numerically, through their electric and magnetic field magnitude calculated by the FEM resolution. Once $Y_{\substack{even\\odd}}$ is calculated, a conversion relation allows us to determine the S-parameter matrix at the discontinuity.

II. NUMERICAL RESULTS

A. *Study case*

The Mode-Matching resolution is tested for a case of a metamaterial's periodic walls. The studied structure is a double transition between a metallic waveguide (PEC) and metamaterial waveguide with a periodic pattern of type 'PIC' (as shown in Figure 3). The PEC waveguide has a radius of $a = 35mm$ and the dimensions of the metamaterial waveguide are given by $A = 72mm$, $a_p = 65.5mm$, $p = 6mm$, $d = 6.5mm$, $w = 5mm$, $\alpha_p = 2°$, $\alpha = 10°$ and $L_M = 54mm$. The surface $\partial\Omega'$, for computing the EM field magnitude, is chosen with a radius of $a' = 63mm$. The elementary cell used in the computation of the hybrides modes and its meshing are presented in Figure 3. The mesh is refined around the 'PIC' pattern in order to accurately mimic the EM field effect on the metamaterial region.

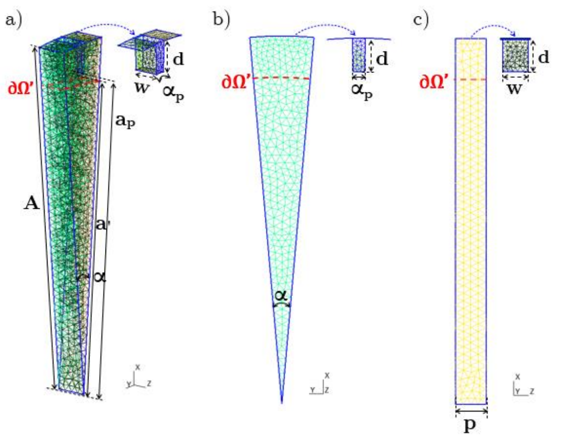

Fig. 4. The elementary cell of the waveguide with periodic PIC walls

B. *S-parameter results*

The double transition is simulated in the TE_{11} single-mode frequency band [2.5GHz,5.22GHz]. The S-parameters are computed by an algorithm MMg based on the MM resolution. To validate our methodology, the results are compared to a full-wave FEM resolution with the Ansys HFSS software. The number of modes of the 'PIC' waveguide used into the MMg algorithm is 12modes. On the other side, the stopping criteria for the HFSS resolution is the variation of S-Parameters with $\Delta S = 0.001$. Thus, the simulated S_{11} parameter (reflection coefficient) and S_{21} parameter (transmission coefficient) are depicted for both resolutions in Figure 4.

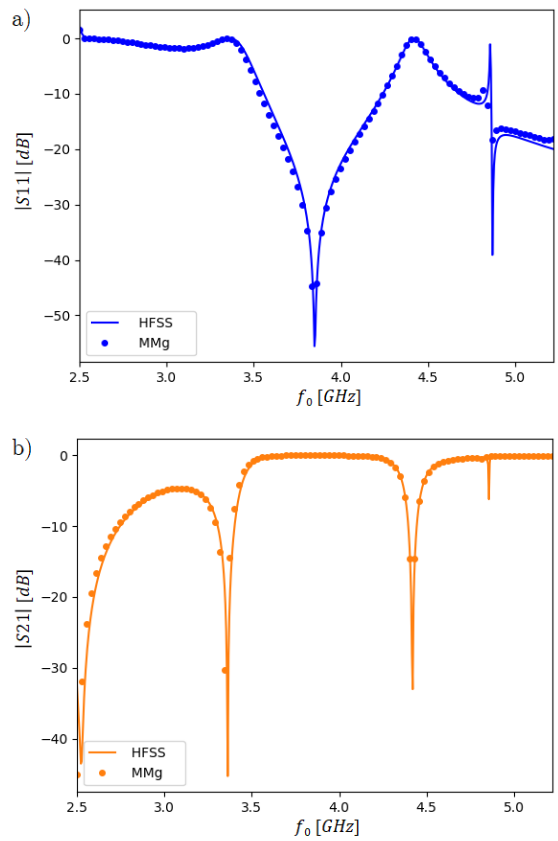

Fig. 4. S-parameter comparison between the MM algorithm and Ansys HFSS: a) |S11|, b) |S21|.

The S-parameters results of MMg and HFSS simulations are in very good agreement. The same resonances and behaviors in reflection and transmission are detected. The concordance of the results, for this case of double transition with a 'PIC' metamaterial wall and other tested cases, confirms the validity of the developed MM resolution.

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