Vector Nodal Meshless Method Applied to Solve the Symmetrical TEAM 13 Problem

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***Abstract*— Traditional meshless methods use scalar-based functions and therefore, present difficulties in approximating vector fields. The Vector Nodal Meshless Method (VNMM) constructs its approximations using shape functions based on the H (curl) spaces and Nédélec's first type elements polynomial space. This paper presents the summary of VNMM for apply in three dimensions vectorial problems, solves the symmetrical TEAM 13 problem and compares the result with one obtained using its full geometry.**

***Keywords*— Meshless methods, tridimensional vector shape functions, vector nodal meshless method**

I. Introduction

Meshless methods are widely known for their versatility to approximate numerically problems that involve movement. However, traditional meshless methods have scalar shape functions and therefore they present difficulties when used to approximate vectorial fields. In order to solve vectorial dynamic problems, the Vector Nodal Meshless Method (VNMM) was developed. The VNMM builds its approximations using shape functions based on **H (curl)** spaces and the Nédélec elements of the first type [1]. In VNMM, a set of nodes is distributed on the studied problem domain and its boundaries, and a unit vector is associated to each node.

The VNMM shape functions have been constructed and used in 3-D problems, specifically in solving the TEAM 13 problem with full geometry [2]. Only half of the geometry needs to be modeled if the symmetry of the problem is considered.

In this paper the VNMM is used for approximate the solution of TEAM 13 problem with half geometry and results are compared with those obtained with the full geometry.

II. Vector Nodal Meshless Method

This section presents the VNMM approximation, its tridimensional shape functions and its main features.

Given a set of nodes distributed on the problem domain Ω in $\mathbb{R}^3$and its boundaries $\partial\Omega$. A unit vector is associated with each node in arbitrary directions as shown in Fig.1.

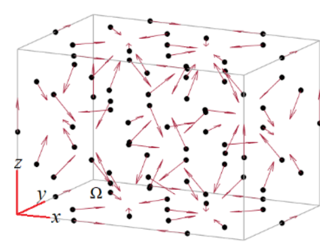

Fig. 1. Distribution of nodes and their vector directions in a domain Ω.

The approximation $\vec{u}^h$ of a given vector field $\vec{u}$ at point $\mathbf{x}$, belonging to Ω, is performed using an arbitrary subset of the nodes. These nodes, called support nodes, and their associated directions will be used to construct the VNMM shape functions. When Ω is composed of more than one material, the support nodes must belong to the same material or be located at the interface between materials. The approximation of the $\vec{u}^h$ is given by [2]:

$$\vec{u}^h(\mathbf{x}) = \sum_{i=1}^{n} \vec{N}_i(\mathbf{x})c_i \tag{1}$$

where n is the number of support nodes, $\vec{N}_i$ are the vector shape functions and c_i are the coefficients of the projection of $\vec{u}^h$ in the directions of the respective nodes. The vector shape function $\vec{N}_i$ associated to the i-th support node is given by [2]:

$$\vec{N}_i = \beta_{1i}\begin{bmatrix}1\\0\\0\end{bmatrix} + \beta_{2i}\begin{bmatrix}0\\1\\0\end{bmatrix} + \beta_{3i}\begin{bmatrix}0\\0\\1\end{bmatrix} + \beta_{4i}\begin{bmatrix}0\\-z\\y\end{bmatrix} + \beta_{5i}\begin{bmatrix}z\\0\\-x\end{bmatrix} + \beta_{6i}\begin{bmatrix}-y\\x\\0\end{bmatrix} \tag{2}$$

where β_{ji}, with $j = 1,\ldots,6$ are the coefficients to be found. The coefficients are found forcing the projection of the i-th shape function in the vector direction associated to the k-th support node to be equal to one if $i = k$, and equal to zero if $i \neq k$.

The VNMM has a convergence rate close to one for approximation of polynomial functions. The method has been tested in waveguides and it correctly compute eigenvalues and its numerical solution is not corrupted by spurious modes.

In terms of memory requirements, VNMM is similar to FEM as the matrices have the same sparsity pattern. However, the VNMM requires more time than FEM to calculate the shape function, since it does not have closed analytic expressions. This time is compensated for by not needing to build a mesh.

III. Problem Definition and Numerical Results

In order to evaluate the performance of the VNMM the TEAM problem 13 is solved [3]. To take account the symmetry only the half of the geometry needs to be modeled.

The behavior of the steel material is nonlinear and defined through a $\vec{B}\vec{H}$ curve shown in [3].

The domain of interest, given by Ω bounded by the surface $\Gamma = \Gamma_g \cup \Gamma_h$, where Γ_g is the boundary with Dirichlet condition and Γ_h is the symmetrical boundary with Neumann condition. The variational form of this problem is: given $\nu(\vec{B}) = \nu_0 \nu_r(\vec{B})$ and $\vec{J}$ find $\vec{A}$ such that [2]:

$$\int_\Omega \left(\nu(\vec{B})\, \nabla \times \vec{A}\right).\left(\nabla \times \vec{F}\right) d\Omega + \epsilon \int_\Omega \vec{A}.\vec{F}\, d\Omega = \int_\Omega \vec{J}.\vec{F} d\Omega \quad (3)$$

where $\vec{A}$ is the magnetic vector potential, $\vec{J}$ is the current density, $\vec{F}$ is the test function and ν is the magnetic . A regularization technique for double curl problems with divergence-free constraint is used by adding the term $\epsilon \int_\Omega \vec{A}.\vec{F}\, d\Omega$ in equation (3). The details of this technique and the choice of ϵ are presented in [4].

The problem is solved using the Vector Nodal Meshless Method, VNMM, with 64,302 nodes distributed in the domain Ω and six support nodes in the approximation. Successive approximations are applied to solve nonlinearity and it converges after 15 iterations with a criterion of 10^{-6}. Table 1 summarizes the computational procedures adopted to obtain the results using the full and the half geometry. Fig. 3 shows the magnitude of the magnetic induction vector $\vec{B}$ in steel excited by a current of 1,000 A-T. Due to the displacement of the steel channels, a three-dimensional magnetic flux density field $\vec{B}$ is generated.

The magnetic flux density is evaluated in the three regions along the steel. Fig. 3 plots the results obtained by VNMM method approximation and the measured data provided by [3]. A good agreement can be observed from these data.

The results shows that the VNMM correctly approximates the magnetostatic problem and its solution is similar to that one obtained with full geometry. However, in the case of the use of the symmetry there is the obvious advantage of the number of unknowns and processing time reductions. In future work, periodic boundary conditions will be introduced and implemented in VNMM, enabling an even greater reduction in the study domain of the TEAM 13 problem.

TABLE I
Summary of Computation Problems

Item	Specification	Specification
Fraction of geometry	1/2	Full
Gauge condition	Regularization technique	
Technique for nonlinear	Successive Approximations	
Approximation method of BH curve	Akima	
Solution for linear equations	Gauss elimination method	
Number of support nodes	6	6
Gauss points for numeric integration	4	4
Number of nodes	64,302	122,018
Number of unknowns	60,046	113,567

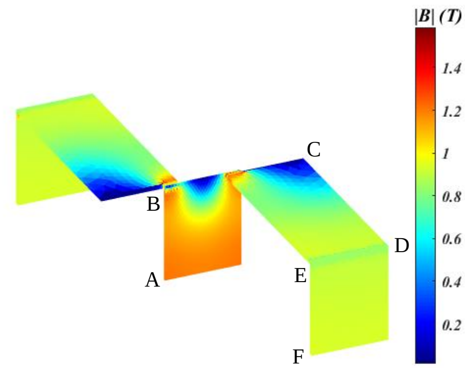

Fig. 2. Magnitude of the magnetic flux density on the surface of the steel structure in TEAM 13 half geometry computed by VNMM.

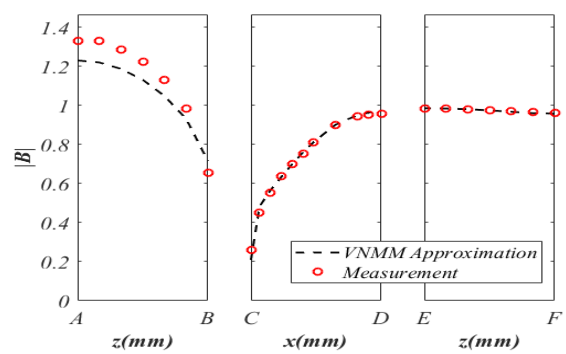

Fig. 3. Numerical results of TEAM problem 13 with 1,000-AT coil current excitation. Magnitude of magnetic flux density in the observation areas of the steel.

Acknowledgement

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References

[1] J. C. Nedelec, "Mixed finite elements in R3," Numerische Mathematik, vol. 35, no. 3, pp. 315–341, 1980.

[2] B. Mara Ferreira Gonçalves, E. J. Silva and R. C. Mesquita, "Vector Nodal Meshless Method for 3-D Applications," in IEEE Transactions on Magnetics, vol. 59, no. 5, pp. 1-4, May 2023.

[3] "Testing electromagnetic analysis methods (T.E.A.M.)," http://www.compumag.org/jsite/team.html, *International Compumag Society*.

[4] H. Duan et. al., "A delta-regularization finite element method for a double curl problem with divergence-free constraint," *SIAM J. Numer. Anal.*, vol. 50, no 6, pp. 3208-3230, 2012.