Multi-fidelity finite-element simulation of transient eddy-current problems with a bias correction on the basis of a recurrent neural network

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***Abstract*—A transient finite-element simulation with a low resolution in space and a high resolution in time is bias-corrected by a recurrent network network, which is trained using the data of a transient finite-element simulation with a high resolution in space and a reduced resolution in time. Upsampling is applied to overcome data sparsity. A coupling with a Gaussian prior is employed to avoid overfitting. The multi-fidelity simulation scheme is successfully applied to a 2D eddy-current model of a quadrupole magnet.**

I. INTRODUCTION

Transient finite element (FE) simulations of models combining a high spatial as well as temporal resolution are computationally expensive. The standard approach to overcome this difficulty within a conventional design process reduces the high-fidelity FE model into a surrogate model, e.g., an electric circuit, a magnetic circuit or a set of state equations, with the aid of, e.g., parameter-extraction techniques, model order reduction [1] of proper orthogonal/generalized decomposition [2], [3]. In a subsequent step, the low-fidelity surrogate model is time-stepped with a sufficiently small time step. This simulation strategies suffers from the approximations made in the model-reduction step, which may unfavourably accumulate in the time-stepping process. Moreover, some reduction techniques may fail to correctly grasp the time-dependent behaviour, especially in the presence of nonlinearities.

In this paper, a high- and low-fidelity model of the same device is co-simulated. The discrepancy between both models is represented by a time-dependent correction term, being a recurrent neural network trained for a limited number of high-fidelity time steps. The merit of this simulation is illustrated for an eddy-current model of a quadrupole magnet.

II. FINITE-ELEMENT MAGNET MODELS

The quadrupole magnet shown in Fig. 1 consists of four coils wound around the four poles of a slightly conducting steel yoke. During ramp-up, affordable, but thermally significant, eddy-current losses occur in the yoke parts, which need to be accounted for during the design process.

The problem is governed by the magnetoquasistatic formulation in terms of the magnetic vector potential $\mathbf{a}(\mathbf{r},t)$, i.e.,

$$\nabla \times (\nu \nabla \times \mathbf{a}) + \sigma \frac{\partial \mathbf{a}}{\partial t} = \mathbf{j}_\text{s}, \qquad \text{in} \quad \Omega; \tag{1a}$$

$$\mathbf{a} = 0, \qquad \text{at} \quad \partial\Omega, \tag{1b}$$

with $\nu(\mathbf{r})$ the reluctivity, $\sigma(\mathbf{r})$ the conductivity, $\mathbf{j}_\text{s}(\mathbf{r},t)$ the current density in the coils, Ω the computational domain and $\partial\Omega$ its boundary.

Moritz von Tresckow acknowledges the support of the German Federal Ministry for Education and Research (BMBF) via the research contract 05K19RDB. Dimitrios Loukrezis and Herbert De Gersem acknowledge the support of the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation), Project-ID 492661287 – TRR 361. The authors acknowledge the financial support of Deutsches Elektronen-Synchrotron DESY.

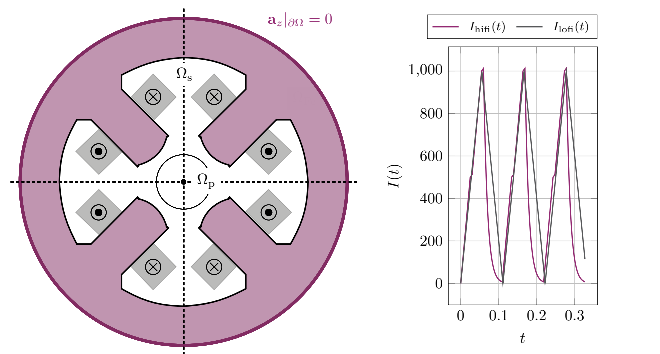

Figure 1. Left: Quadrupole magnet with the iron yoke Ω_Fe, the domain with current excitation Ω_s and the air domain Ω_p. Right: Current excitations $I_\text{hifi}(t)$ and $I_\text{lofi}(t)$ for the high- and low-fidelity model, respectively.

The formulation is discretized by the Ritz-Galerkin FE method, whereby the translatory symmetry is coded into the edge shape functions $\mathbf{w}_j(\mathbf{r}) = w_j(x,y)\,\mathbf{e}_z$, with $w_i(x,y) \in H_0(\text{grad};\Omega)$ for discretizing $\mathbf{a}$, i.e.,

$$\mathbf{a}(\mathbf{r},t) = \sum_{j=1}^{N_\text{dof}} \hat{a}_j(t)\,\mathbf{w}_j(\mathbf{r}), \tag{2}$$

with $\hat{a}_j(t)$ the N_dof degrees of freedom (dof), associated with the nodes of the 2D mesh. Using the backward Euler method for time discretization, the discrete FE problem reads

$$(\Delta t\,\mathbf{A} + \mathbf{M})\,\hat{\mathbf{a}}_{t_{k+1}} = \Delta t\,\mathbf{b}\,(t_{k+1}) + \mathbf{M}\,\hat{\mathbf{a}}_{t_k}, \tag{3}$$

with $\mathbf{A}$ and $\mathbf{M}$ the stiffness and mass matrix and $\mathbf{b}$ the load vector.

Two FE models with different resolutions are set up. The low-fidelity model is simulated for the excitation current $I_\text{lofi}(t)$ with $N_\text{dof}^\text{lofi} = 895$ dofs and $N_T^\text{lofi} = 327$ time steps, whereas the high-fidelity model is simulated for the excitation current $I_\text{hifi}(t)$ with $N_\text{dof}^\text{hifi} = 277\,594$ dofs and $N_T^\text{lofi} = 79$ time steps (Fig. 1). The respective solution vectors are denoted by $\hat{\mathbf{a}}_{t_{k+1}}^\text{lofi}$ and $\hat{\mathbf{a}}_{t_{k+1}}^\text{hifi}$, respectively.

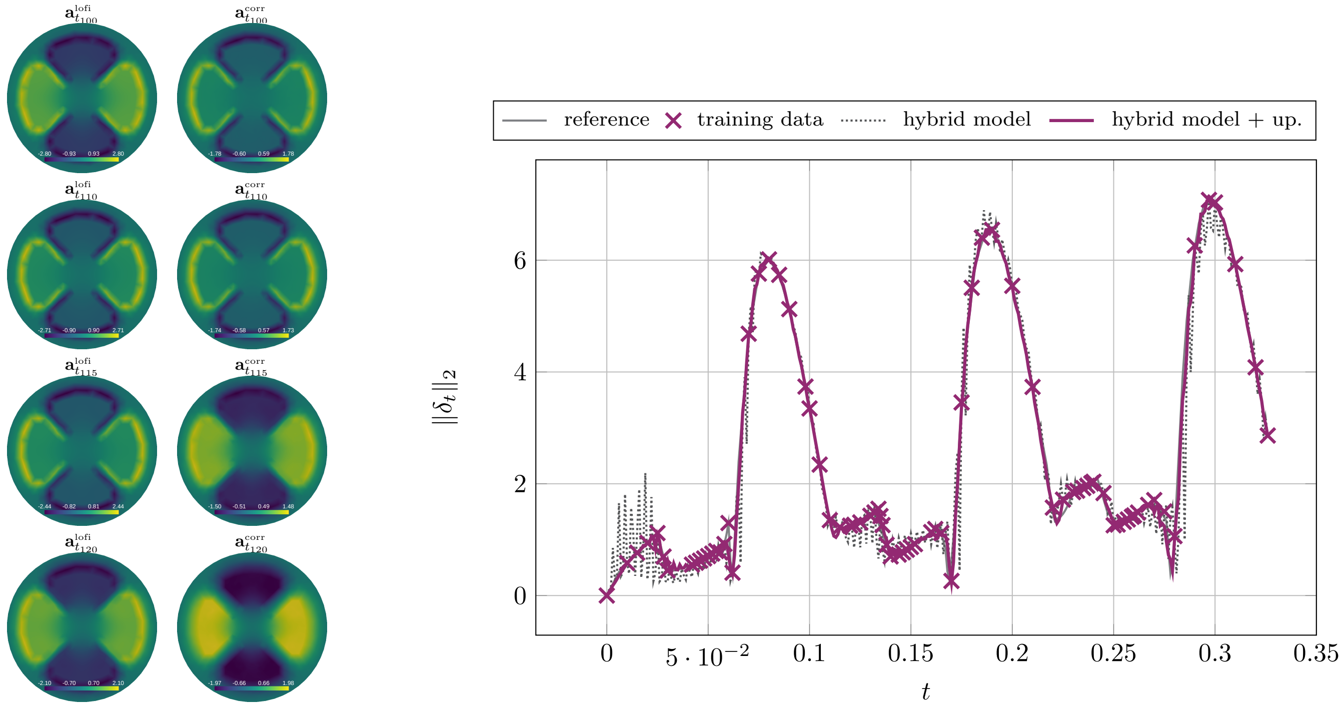

Figure 2. Left: Low-fidelity and bias-corrected solutions for the magnetic vector potential. Right: Spatially-integrated discrepancy function over time.

III. MULTI-FIDELITY MAGNET MODEL

The solutions of the low- and high-fidelity magnet models are put in relation to each other by

$$\mathbf{a}^{\text{hifi}}(\mathbf{r},t) = \mathbf{a}^{\text{lofi}}(\mathbf{r},t) + \boldsymbol{\delta}(\mathbf{r},t), \tag{4}$$

where $\boldsymbol{\delta}(\mathbf{r},t) : \mathbb{R}^3 \to \mathbb{R}^3$ is a discrepancy function capturing the systematic errors between both models [4]. In the discrete setting, the high-fidelity data $\mathbf{A}_{\text{hifi}} := \left\{\mathbf{a}^{\text{hifi}}(\mathbf{r},t_k)\right\}_{t_k \in T_{\text{hifi}}}$ is approximated in space according to (2) with shape functions $\mathbf{w}_j^{\text{hifi}}(\mathbf{r})$ and is only known at the time instants $t_k \in T_{\text{hifi}} = \left\{t_0^{\text{hifi}}, \ldots, t_{N_T^{\text{hifi}}}^{\text{hifi}}\right\}$. Hence, to correct on the low-fidelity mesh and for all low-fidelity time steps $t_l \in T_{\text{lofi}} = \left\{t_0^{\text{lofi}}, \ldots, t_{N_T^{\text{lofi}}}^{\text{lofi}}\right\}$, a correction of the form

$$\boldsymbol{\delta}_{t_l^{\text{lofi}}}(\mathbf{r}) = \sum_{j=1}^{N_{\text{dof}}^{\text{lofi}}} \hat{\boldsymbol{\delta}}_{j,t_l^{\text{lofi}}}\, \mathbf{w}_j(\mathbf{r}), \tag{5}$$

is needed [5]. The key point in this paper is to use a recurrent neural network (RNN) to produce the coefficients $\hat{\boldsymbol{\delta}}_{j,t_l^{\text{lofi}}}$ in (5). The RNN is trained according to the discrepancy data

$$\mathcal{D} := \left\{\mathcal{T}\left(\mathbf{a}_{t_k}^{\text{hifi}}\right) - \mathbf{a}_{t_k}^{\text{lofi}}\right\}_{t_k \in T_{\text{hifi}}}, \tag{6}$$

where $\mathcal{T}$ is an operator projecting a solution from the high-fidelity onto the low-fidelity mesh. To account for the data sparsity of the high-fidelity model, data upsampling is applied [6]. Moreover, overfitting is prevented by coupling the artificial high-fidelity solutions with a Gaussian prior [6]. Finally, the bias-corrected multi-fidelity solution reads

$$\mathbf{a}^{\text{mufi}}(\mathbf{r},t_l^{\text{lofi}}) = \mathbf{a}^{\text{lofi}}(\mathbf{r},t_l^{\text{lofi}}) + \boldsymbol{\delta}_{t_l^{\text{lofi}}}(\mathbf{r}). \tag{7}$$

IV. RESULTS & CONCLUSION

The low-fidelity and bias-corrected solutions for the magnetic vector potential are shown in Fig. 2 at four time instants. The errors of both solutions compared to the fully resolved high-fidelity model are $39.847\,\%$ and $0.613\,\%$, respectively, which illustrates the power of the bias correction along the RNN. The multi-fidelity model resolves the spatial dependencies by the FE method and the time dependencies by the RNN, which turns out to be beneficial, at least for the considered problem class. The bias correction allows magnetodynamic FE simulations, essentially with a low-fidelity model, even with a limited data set coming from the high-fidelity model.

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