Adjoint sensitivity method for nonlinear transient electrothermal finite-element models

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***Abstract*—An adjoint sensitivity formulation is set up for highly-nonlinear transient electrothermal models including field-grading materials with a field- and temperature-dependent behaviour. The technique allows to efficiently calculate the sensivities of quantities of interest with respect to a large number of design parameters, including parameters defining the field-grading material properties.**

I. INTRODUCTION

Avoiding insulation failures in high-voltage (HV) devices requires electric field strengths that stay below breakdown thresholds at all points in space. This is accomplished by avoiding sharp edges of metallic parts and by a meticulous design of insulating parts, which includes an appropriate choice of dielectric materials and their geometries. An additional measure is taken by applying resistive field grading, i.e., shaping the electric field distribution by so-called *field-grading materials* (FGMs) that become increasingly conducting beyond a threshold electric field strength. The high nonlinearity and the temperature dependence of FGMs (Fig. 1), however, substantially complicate the numerical simulation of such devices [2]. FGMs can be tailored to the application, which at one side offers unprecedented design options, but unfavourably increases the number of design parameters.

II. ELECTROTHERMAL FINITE-ELEMENT MODEL

The electrothermal field problem is described by [3]

$$-\nabla\cdot(\sigma\nabla\Phi)-\nabla\cdot\frac{\partial}{\partial t}(\varepsilon\nabla\Phi)=0; \qquad (1a)$$

$$-\nabla\cdot(\lambda\nabla\vartheta)+\frac{\partial}{\partial t}(c_\mathrm{V}\vartheta)=\dot{q}_\mathrm{Joule}, \qquad (1b)$$

with $\Phi(\mathbf{r},t)$ the electric scalar potential, $\vartheta(\mathbf{r},t)$ the temperature, $\sigma(\mathbf{r},\mathbf{E},\vartheta)$ the electric conductivity, $\varepsilon(\mathbf{r},\mathbf{E},\vartheta)$ the permittivity, $\lambda(\mathbf{r},\vartheta)$ the thermal conductivity, $c_\mathrm{V}(\mathbf{r},\vartheta)$ the volumetric heat capacity, $\dot{q}_\mathrm{Joule}=\mathbf{J}\cdot\mathbf{E}$ the Joule loss density, $\mathbf{J}(\mathbf{r},t)=\sigma\mathbf{E}$ the current density and $\mathbf{E}(\mathbf{r},t)$ the electric field strength. Φ and ϑ are discretised by nodal finite-element (FE) shape functions on the domain Ω and resolved in time by the backward Euler method on the time interval $[t_0,t_\mathrm{f}]$.

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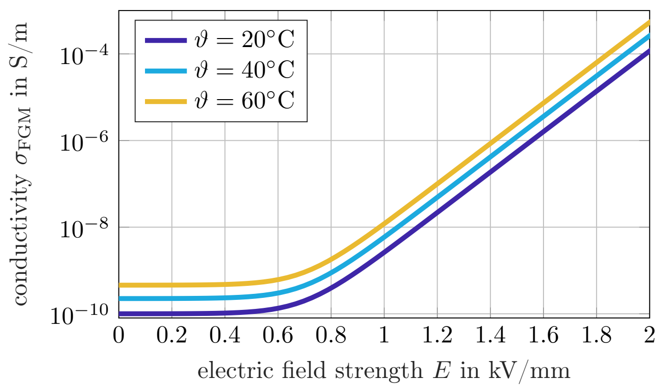

Figure 1. Dependence of the conductivity of a field-gradient material (FGM) on the electric field strength and the temperature.

III. DESIGN CONTEXT

The design parameters are denoted by p_j, $j=1,\dots,N_\mathrm{par}$ with N_par the number of design parameters. The quantities or interest (QoIs) are expressed by

$$q_i=\int_\Omega\int_{t_0}^{t_\mathrm{f}} g_i(\mathbf{r},t,\Phi,\vartheta)\,\mathrm{d}\Omega\,\mathrm{d}t, \qquad (2)$$

$i=1,\dots,N_\mathrm{qoi}$, with N_qoi the number of QoIs and $g_i(\mathbf{r},t,\Phi,\vartheta)$ the post-processing kernels. As an example, the Joule heat accumulated from t_0 to t_f is modelled by $g_\mathrm{heat}=\mathbf{J}\cdot\mathbf{E}$. Uncertainty quantification, optimization and engineering decisions heavily rely upon the sensivities $S_{ij}=\frac{\partial q_i}{\partial p_j}$ of the QoIs with respect to the design parameters.

IV. ADJOINT FORMULATION

In *direct sensitivity* methods, the derivatives $\frac{\mathrm{d}\Phi}{\mathrm{d}p_j}$ and $\frac{\mathrm{d}\vartheta}{\mathrm{d}p_j}$ are computed from the derivative of formulation (1). Then, S_{ij} is found by applying the chain rule to (2). This approach, however, scales unfavourably with N_par, as is the case here.

The *adjoint sensitivity* method follows from adding (1a) multiplied by $\eta_i(\mathbf{r},t)$ and (1b) multiplied by $\xi_i(\mathbf{r},t)$ into the integral of (2). Then, after partial integration in space and time, one observes that calculating $\frac{\mathrm{d}\Phi}{\mathrm{d}p_j}$ and $\frac{\mathrm{d}\vartheta}{\mathrm{d}p_j}$ can be avoided for an appropriate choice of adjoint solutions (η_i,ξ_i), $i=1,\dots,N_\mathrm{qoi}$, computed from the adjoint problems

$$-\nabla\cdot\left(\overline{\overline{\sigma}}_\mathrm{d}\nabla\eta_i\right)+\nabla\cdot\left(\overline{\overline{\varepsilon}}_\mathrm{d}\frac{\partial}{\partial t}\nabla\eta_i\right)$$
$$+\nabla\cdot\left(\left(\overline{\overline{\sigma}}_\mathrm{d}\mathbf{E}+\mathbf{J}\right)\xi_i\right)=\frac{\partial g_i}{\partial\Phi}; \qquad (3a)$$

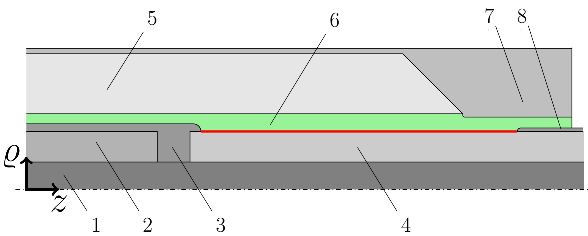

Figure 2. Cross section of a HVDC cable joint with copper conductor (1), aluminum connector (2), conductive silicone rubber (3), cross-linked polyethylene (4), insulating silicone rubber (5), nonlinear field-grading material (6), outer aluminum body (7) and outer cable semiconductor (8).

$$-\nabla\cdot(\lambda\nabla\xi_i)-\left(c_{\mathrm{V}}+\frac{\partial c_{\mathrm{V}}}{\partial\vartheta}\vartheta\right)\frac{\partial\xi_i}{\partial t}+\frac{\partial\lambda}{\partial\vartheta}\nabla\vartheta\cdot\nabla\xi_i$$
$$-\frac{\partial\sigma}{\partial\vartheta}E^2\xi_i-\frac{\partial\sigma}{\partial\vartheta}\mathbf{E}\cdot\nabla\eta_i+\frac{\partial\varepsilon}{\partial\vartheta}\mathbf{E}\cdot\frac{\partial}{\partial t}\nabla\eta_i=\frac{\partial g_i}{\partial\vartheta},\qquad(3b)$$

$i=1,\dots,N_{\mathrm{qoi}}$, where $\overline{\overline{\sigma}}_{\mathrm{d}}(\mathbf{r},t)$ and $\overline{\overline{\varepsilon}}_{\mathrm{d}}(\mathbf{r},t)$ are the differential conductivity and permittivity, and where all materials and fields are evaluated for the nominal solution (Φ_0,ϑ_0). A full derivation of (3) is given in [4]. The adjoint formulation is linear, incorporates a tight coupling between the adjoint fields η_i and ξ_i and needs reverse time-stepping. The adjoint procedure is particularly efficient when $N_{\mathrm{qoi}}\ll N_{\mathrm{par}}$ because only N_{qoi} adjoint problems needs to be solved instead of N_{par} derived problems [5].

V. HIGH-VOLTAGE DC CABLE JOINT

The adjoint sensitivity approach is applied to a 2D axisymmetric FE model of a HVDC cable joint [1], [6] (Fig. 2). During an overvoltage event due to switching, which doubles the excitation voltage within $1\,\mathrm{ms}$ and declines over $15\,\mathrm{ms}$, the electric field along the FGM changes as shown in Fig. 3. The FGM prevents the electric field strength to exceed the critical value $E_{\mathrm{crit}}=2\,\mathrm{kV/mm}$ (light blue curve for $t=0.357\,\mathrm{ms}$). The field-grading effect is exerted by the rise of the conductivity of the FGM, which comes together with an increase in temperature. The latter is obvious from comparing the electric fields at $t=0\,\mathrm{ms}$ (dark blue line) and $t=30\,\mathrm{ms}$ (orange line), at which the overvoltage situation has vanished. These effects illustrate the strong nonlinearities and temperature-dependencies of FGMs frequently used in HVDC cable joints.

Future designs of HVDC cable joints may benefit from new material processing techniques that allow to produce FGMs with tailored properties [7]. The conductivity of a generic FGM can be represented by

$$\sigma_{\mathrm{FGM}}(E,\vartheta)=p_1\frac{1+p_4^{\frac{E-p_2}{p_2}}}{1+p_4^{\frac{E-p_3}{p_2}}}e^{-p_5\left(\frac{1}{\vartheta}-\frac{1}{\vartheta_0}\right)},\qquad(4)$$

with ϑ_0 a reference temperature and $\{p_1,\dots,p_5\}$ the design parameters shaping the material characteristic [1]. The adjoint sensitivity formulation allows to calculate the sensitivities of the Joule heat accumulated during the overvoltage event with respect to $\{p_1,\dots,p_5\}$ along one solution of the forward nonlinear transient electrothermal problem (1) and one solution of the adjoint backward linear transient problem (3) (Fig. 4).

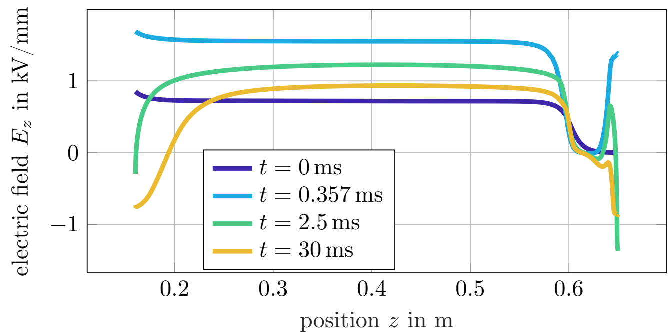

Figure 3. Electric field strength in the FGM, tangentially to the interface between FGM and cross-linked polyethylene (along the red line in Fig. 2).

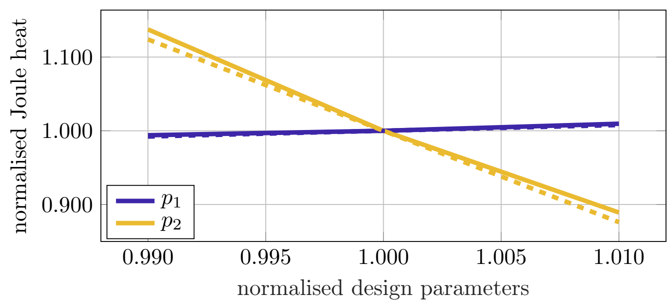

Figure 4. Sensitivities of the Joule heat accumulated during the overvoltage event with respect to p_1 and p_2 (visualised as dashed lines in the nominal point, which is normalised to $(1,1)$); Joule heat as a function of p_1 and p_2 calculated by parameter variations for reason of validation (solid lines).

VI. CONCLUSION

The adjoint sensitivity approach allows an efficient computation of the sensitivities of a few QoIs with respect to a larger number of design parameters for complicated nonlinear transient electrothermal problems.

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